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Experimental identification of damping

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Abstract

Four different procedures for identification of damping in mechanical systems are utilized. The procedures are based on results of computational simulations of mathematical models obtained through the finite element method—FEM and experimental procedures based on modal analysis. The rational fraction polynomial method (RFPM) is used to modal analysis. The validation of the numeric models is done through confrontations of computational results and experimental data obtained by accelerometers or proximity sensors. Procedures for reduction of the numeric models are used due to the fact that only few points are used to obtain the experimental values. Three different mechanical systems are analyzed: system of two degrees-of-freedom (two bars in balancing), thin plate clamped in an extremity and free in the other, and electric transmission line cable submitted to an axial load.

Keywords: Damping; Identification; Reduced model; Modal analysis

1. Introduction

The correlation between analytic data and experimental results introduces inherent difficulties in the obtainment of the same. When vibration tests are conducted, several source errors can be present: incorrect calibration of the equipment, excessive noise, damaged equipment, incorrect interpretation of the data, incorrect location of sensor, etc. Analytic finite element models also can contain errors: incorrect concepts of modeling, uncertainties in the properties of the materials, insufficient details of modeling, incorrect boundary condition, etc. When the analytic results are different from experimental data, the FE model should be corrected or updated to present good concordance between analytic and experimental values. Thus, the updating models can be considered as the best dynamic representation of a structure. The corrections of the mathematical models are done through the data processing of structural tests and comparisons with the obtained simulated values.

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Friswell et al. (1995, 1998) used an updating method for reduced systems (IRS—improved reduced system), which allows estimating modal parameters of a system with good precision. The reduction of the model is done through dynamic procedures. A iterative process allows the convergence for a reduced model that reproduces a modal set of the complete system.

In the application of reduced models, a limiting factor is the adequate procedure to describe the system damping matrix. In this sense, Pilkey (1998) describes two types of procedures, direct and iterative, for computation of the system damping matrix. The procedures are based on previous knowledge of the stiffness and inertia matrix and the experimental modal behavior of the system.

Adhikari (2000) describes some procedures for the analysis and damping estimation of structural systems. One of them it uses the damping ratios and the corresponding natural frequencies to estimate a function. Another technique needs just the natural frequencies and the vibrating complexes modes for identification of the damping matrix.

The central point of all the updating methods of the damping matrix, it is the modal analysis of the system. In this sense, Iglesias (2000) uses several techniques to estimate the modal parameters: complex exponential method (CEM), Ibrahim team domain (ITD), rational fraction polynomial method (RFPM) and Hilbert envelope method (HEM).

Usually in the updating models are used incomplete sets of experimental vibrating modes due to physical limitation in the position and quantity of sensors used in the obtainment of the experimental data. This limitation finishes for inserting errors to the systems. These errors can modify the characteristics of system. To reduce the effects of a bad measurement, the adequate choice of the sensors position is the fundamental importance.

Reynier and Abou-Kandil (1999) propose two methods to determine the best sensors location. One method is based on minimizing of the measurement noise and the other analyze the vibrating modes.

Lamarque et al. (2000) introduce a wavelet-based method which is similar to decrement logarithmic formulation to estimate the damping ratio on time domain. The comparisons between theoretical and numeric values present excellent results, when the signal-noise relationship is good.

Ruotolo and Storer (2001) describe a method (GST—global smoothing technique) of modal analysis generally used in systems with high modal density, which uses the measured frequency response function.

Bosse et al. (1998) describe the theory and the computational implementation of an algorithm developed for the identification of the online modal parameters. This method captures the modal behavior of the structure in real time.

Rad (1997) presents good material about updating models, which describes and evaluates the main methods used in the updating of numeric models of dynamic structures.

Barbieri et al. (2004a) present a theoretical and experimental study about the dynamical behavior of transmission cables. The rational fraction polynomial method (RFPM) was used to the identification of the modal parameters. Reduced models were used to estimate the structural damping matrix.

Barbieri et al. (2004b) establish a procedure for damping identification in transmission line cables systems computing of a simple way the damping matrix. The authors used different procedures for the matrix damping estimation.

In this work, the authors, try to adjust the damping matrix for three different systems: system of two degrees-of-freedom (two bars in balancing), thin plate clamped in an extremity and free in the other, and electric transmission line cable submitted to an axial load. Four different procedures for identification of damping in mechanical systems are utilized. The procedures are based on results of computational simulations of mathematical models obtained through the finite element method—FEM and experimental procedures based on modal analysis for parameters identification. Two procedures are based in the methods described by Pilkey (1998) and two procedures are based in the methods described by Adhikari (2000).

2. Mathematical model

The equations of motion of an damped system can be obtained through finite element method and gives:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damper and stiffness matrices; $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are the acceleration, velocity and displacement vectors and $\mathbf{f}(t)$ is the excitation vector. Usually, the values of the elements of $\mathbf{M}$ and $\mathbf{K}$ are well-known. However, the elements of $\mathbf{C}$ are of difficult obtainment. To estimate the values of the elements of $\mathbf{C}$ several procedures based on knowledge of the finite element or analytical mass and stiffness matrices and measured eigendata have been developed. To find the modal parameters several methods are used (Iglesias, 2000): the rational fraction polynomial (RFP) method, the prony or complex exponential method (CEM), the Ibrahim time domain (ITD) and the Hilbert envelope method. All the methods are based on frequency response function (FRF) between an excitation signal and the measured parameter:

$$\begin{bmatrix} X_1(\omega) \\ \cdot \\ \cdot \\ X_n(\omega) \end{bmatrix} = \begin{bmatrix} h_{11}(\omega) & \cdot & \cdot & h_{1n}(\omega) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ h_{n1}(\omega) & \cdot & \cdot & h_{nn}(\omega) \end{bmatrix} \begin{bmatrix} F_1(\omega) \\ \cdot \\ \cdot \\ F_n(\omega) \end{bmatrix} \tag{2}$$

where ω is the frequency; X_i is the variable value in point i; h_{ij} is the value of the FRF matrix element; F_i is the value of the force applied at point i.

An iterative procedure described in Friswell et al. (1995) can be used to adjustment of a reduced system of equations. The procedure is based on the partitioned equation (1) considering the damping matrix null:

$$\begin{bmatrix} \mathbf{M}_{\mathrm{mm}} & \mathbf{M}_{\mathrm{ms}} \\ \mathbf{M}_{\mathrm{sm}} & \mathbf{M}_{\mathrm{ss}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_{\mathrm{m}} \\ \ddot{\mathbf{x}}_{\mathrm{s}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{mm}} & \mathbf{K}_{\mathrm{ms}} \\ \mathbf{K}_{\mathrm{sm}} & \mathbf{K}_{\mathrm{ss}} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{\mathrm{m}} \\ \mathbf{x}_{\mathrm{s}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{\mathrm{m}} \\ 0 \end{Bmatrix} \tag{3}$$

where $\mathbf{M}$ is the inertia matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{x}$ is the displacement vector, $\mathbf{f}$ is the excitation vector, m are the measured nodal points and s are the non-measured nodal points. The reduced system is

$$\mathbf{M}_{\mathrm{r}}\ddot{\mathbf{x}}_{\mathrm{m}} + \mathbf{K}_{\mathrm{r}}\mathbf{x}_{\mathrm{m}} = \mathbf{f}_{\mathrm{m}} \tag{4}$$

where $\mathbf{M}_{\mathrm{r}}$ and $\mathbf{K}_{\mathrm{r}}$ are the reduced matrices. In this study was considered the first five mode shapes obtained through five accelerometers.

2.1. Method 1

With the estimated values of the damping ratio and the natural frequencies of the three selected modes shape, it is possible to obtain a function of the form (Adhikari, 2000):

$$\xi(\omega) = a_0 + a_1\omega + a_2\omega^2 \tag{5}$$

The damping matrix can be obtained as

$$\mathbf{C} = 2\mathbf{M}_{\mathrm{r}}\sqrt{\mathbf{M}_{\mathrm{r}}^{-1}\mathbf{K}_{\mathrm{r}}}\left[a_0\mathbf{I} + a_1\left(\mathbf{M}_{\mathrm{r}}^{-1}\mathbf{K}_{\mathrm{r}}\right)^{1/2} + a_2\left(\mathbf{M}_{\mathrm{r}}^{-1}\mathbf{K}_{\mathrm{r}}\right)^{2/2}\right] \tag{6}$$

2.2. Method 2

Another procedure to find the damping matrix $\mathbf{C}$ is described in Adhikari (2000). This procedure is based on the estimation of the modal damping matrix:

$$C'_{kj} = \frac{\left(\hat{\omega}_j^2 - \hat{\omega}_k^2\right) B_{kj}}{\hat{\omega}_j}, \quad k \neq j \tag{7}$$

and

$$C'_{jj} = 2\Im(\hat{\lambda}_j) \tag{8}$$

where $\hat{\lambda}_j$ is the complex natural frequency, $\hat{\omega}_j$ is the undamped natural frequency (real part of $\hat{\lambda}_j$), $\Im(\hat{\lambda}_j)$ is the imaginary part of $\hat{\lambda}_j$, $\mathbf{B}$ is an auxiliary matrix. The damping matrix is

$$\mathbf{C} = \left[(\widehat{\mathbf{U}}^{\mathrm{T}}\mathbf{U})^{-1}\widehat{\mathbf{U}}^{\mathrm{T}}\right]^{\mathrm{T}} \mathbf{C}' \left[(\widehat{\mathbf{U}}^{\mathrm{T}}\mathbf{U})^{-1}\widehat{\mathbf{U}}^{\mathrm{T}}\right] \tag{9}$$

where $\mathbf{U}$ is the real part of the complex mode shape matrix.

2.3. Methods 3 and 4

Two procedures to matrix damping identification are described by Pilkey (1998), iterative and direct damping identification. The iterative method begins with the initial choose of damping matrix C_0 and the eigenvectors normalization as

$$\boldsymbol{\phi}_i^{\mathrm{T}}(2\mathbf{M}_{\mathrm{r}}\lambda_i + \mathbf{C}_{m-1})\boldsymbol{\phi}_i = 1 \tag{10}$$

where m is the variable increment, $\boldsymbol{\phi}_i$ is the eigenvector and λ_i is the eigenvalue. The damping matrix is

$$\mathbf{C}_m = -\mathbf{M}_{\mathrm{r}}(\mathbf{\Phi}\mathbf{\Lambda}^2\mathbf{\Phi}^{\mathrm{T}} + \overline{\mathbf{\Phi}\mathbf{\Lambda}^2}\mathbf{\Phi}^*)\mathbf{M}_{\mathrm{r}} \tag{11}$$

where the overbar represents the complex conjugate and * represents the complex conjugate transpose; $\mathbf{C}_m$ is the damping matrix after m iterations, $\mathbf{\Phi}$ is a matrix of eigenvectors and $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues. The new damping matrix (Eq. (11)) is substituted into Eq. (10) until the convergence has been obtained.

The direct method normalize the eigenvectors as

$$\boldsymbol{\phi}_i^{\mathrm{T}}\left(\mathbf{M}_{\mathrm{r}}\boldsymbol{\lambda}_i^2 - \mathbf{K}_{\mathrm{r}}\right)\boldsymbol{\phi}_i = \boldsymbol{\lambda}_i \tag{12}$$

The damping matrix is obtained by Eq. (11).

3. Physical models

The first model is a system of two degrees-of-freedom (two bars in balancing). The Fig. 1 shows schematically the main components of the system and the equipment used to data acquisition.

The system is composed by an extremely rigid structure (1) in which is coupled, by means of connections (6) that only allow rotation, two bars (5) and two springs (4). The accelerometers (2) are fixed in points located in the bars and the systems excitation is done directly through an impact hammer (3). The physical system data are: mass of the bar 1(upper side) = 1.941 kg; mass of the bar 2 = 1.016 kg; stiffness of the spring 1 (upper side) = 3900 N/m; stiffness of the spring 2 = 900 N/m; length of the bar 1 = 0.75 m and length of the bar 2 = 0.49 m.

A thin plate clamped in an extremity and free in the other composes the second model. The Fig. 2 shows schematically the main components of the system and the equipment used to data acquisition. The system is composed by an extremely rigid structure (1) in which is coupled by means of a clamp (5), a plate (4). The vibration sensors (2) are placed nearby (without contact) of the points located in the plate and the exci-

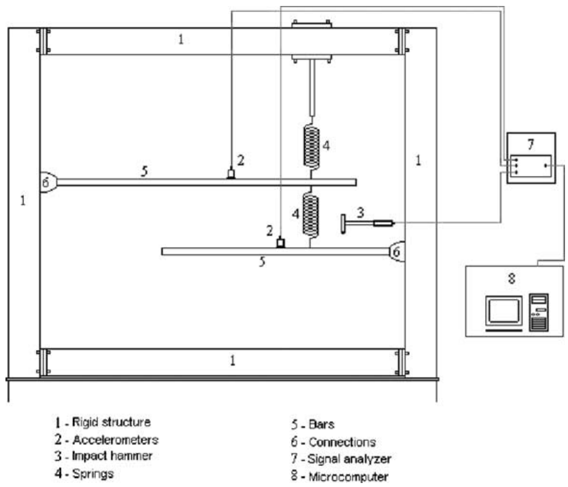

Fig. 1. Two degrees-of-freedom system.

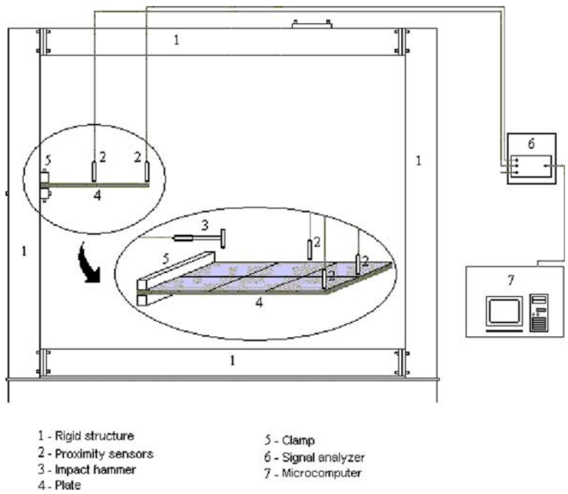

Fig. 2. Clamped plate.

tation is done directly through an impact hammer (3). The physical system data are: steel, dimensions: $31\times19.5\times2$ mm; elastic modulus = 210 GPa; density = 7904.06 kg/m^3.

The third model is a transmission line cable submitted to an axial load. The Fig. 3 shows schematically the main components of the system and the equipment used to data acquisition. One microcomputer controls the testing performance through the use of an intelligent interface (e1) and on-line monitoring of the signals of one load sensor (e7). The mechanical pre-tensioning system (e3–e5) applies sufficient load (5% RTS—conductor rated tensile strength) only to put in place the sensors. The load capacity of this system is 10 kN.

The system (e7–e12) serves to control automatically the mechanical load in the cable by using a servomechanism. The control signal is proportional to the difference between the programmed load and the load measured by the load cell. The load capacity of this system is 200 kN. Two load cells can be used with

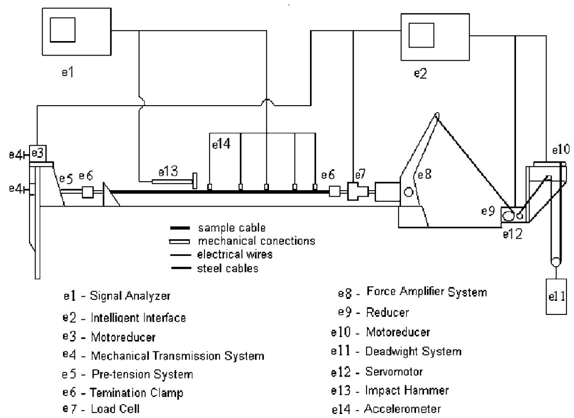

Fig. 3. Schematic view of the testing system for overhead line cables.

load capacity of 200 kN with resolution of 96 N and the 50 kN with resolution of 25 N. The external load is applied to the system through an impact hammer (e13). The vibratory signals are acquired through three accelerometers (e14) placed along half the sample. The signals are collected and manipulated in a signal analyzer (e1). The cable used was the Ibis type whose parameters are: specific mass 0.8127 kg/m; rigidity flexural (EI) 11.07 Nm^2. For the modal identification were placed three accelerometers in the cable, in the positions L/2, 3L/8, L/4. The sample length was 32.322 m.

4. Results

To the methods 1 and 2 the real part of the eigenvectors were mass-normalized through the reduced modal and mass matrices:

$$\mathbf{\Phi}_r^T \mathbf{M}_r \mathbf{\Phi}_r = \mathbf{I} \tag{13}$$

where $\mathbf{\Phi}_r$ is the reduced matrix of eigenvectors, $\mathbf{M}_r$ is the reduced mass matrix and $\mathbf{I}$ is the identity matrix. The same procedure was used to normalize the imaginary part of the eigenvectors. Eqs. (10) and (12) shows the eigenvectors normalization to the methods 3 and 4.

The modal results obtained for the model shown in Fig. 1 are represented in Table 1. The numeric values are obtained through a mathematical undamped system with two degrees-of-freedom. The experimental values are obtained through the rational fraction polynomial method (RFPM).

The damping matrices adjusted with the four different methods are represented in Tables 2 and 3. The frequencies and the damping ratios were calculated considering the adjusted damping matrices.

Table 1
Modal parameters (two bars in balancing model)

	Mode number	Frequency [Hz]	Damping ratio
Numeric	1	2.3473	
RFPM		2.3907	0.0264
Numeric	2	10.4169	
RFPM		10.4150	0.0052

Table 2
Damping matrix adjusted for different methods considering only the real part of the eigenvectors (two bars in balancing model)

Method	Damping matrix	Frequency [Hz]	Damping ratio
1	$C = \begin{bmatrix} 0.3384 & 0.0023 \\ 0.0023 & 0.0662 \end{bmatrix}$	2.3464 10.4167	0.0264 0.0052
2	$C = \begin{bmatrix} 0.3320 & -0.0178 \\ -0.0178 & 0.0697 \end{bmatrix}$	2.3464 10.4167	0.0271 0.0053
3	$C = \begin{bmatrix} 0.3596 & 0.0230 \\ 0.0230 & 0.0649 \end{bmatrix}$	2.3464 10.4167	0.0268 0.0054
4	$C = \begin{bmatrix} 0.3515 & 0.0215 \\ 0.0215 & 0.0616 \end{bmatrix}$	2.3465 10.4167	0.0254 0.0053

Table 3
Damping matrix adjusted for different methods considering the complex eigenvectors (two bars in balancing model)

Method	Damping matrix	Frequency [Hz]	Damping ratio
1	$C = \begin{bmatrix} 0.3384 & 0.0023 \\ 0.0023 & 0.0662 \end{bmatrix}$	2.3464 10.4167	0.0264 0.0052
2	$C = \begin{bmatrix} 0.3217 & -0.0526 \\ -0.0526 & 0.0762 \end{bmatrix}$	2.3464 10.4166	0.0283 0.0054
3	$C = \begin{bmatrix} 0.2713 & -0.0231 \\ -0.0231 & 0.0741 \end{bmatrix}$	2.3463 10.4168	0.0286 0.0044
4	$C = \begin{bmatrix} 0.3112 & -0.0071 \\ -0.0071 & 0.1014 \end{bmatrix}$	2.3454 10.4167	0.0400 0.0049

The modal results obtained for the model shown in Fig. 2 are represented in Table 4. The numeric values are obtained through computational simulation of a FEM model with order of 105×105. The experimental values are obtained through the rational fraction polynomial method (RFPM).

As it was used three proximity sensors in the positions showed in Fig. 2 the reduced system has order of 3×3. To create the mass and stiffness matrices, the iterated improved reduced system (iterated IRS) method was used. The Tables 5 and 6 show the damping matrices estimated with different methods.

The modal results obtained for the model shown in Fig. 3 are represented in Table 7. The numeric values are obtained through computational simulation of a FEM model with order of 35×35. The experimental values are obtained through the rational fraction polynomial method (RFPM).

Table 4
Modal parameters (clamped plate model)

	Mode number	Frequency [Hz]	Damping ratio
Numeric	1	17.592	
RFPM		16.8969	0.0044
Numeric	2	63.814	
RFPM		61.9032	0.0011
Numeric	3	110.84	
RFPM		105.521	0.0025

Table 5
Damping matrix adjusted for different methods considering only the real part of the eigenvectors (clamped plate model)

Method	Damping matrix	Frequency [Hz]	Damping ratio
1	$C = \begin{bmatrix} 0.7821 & -0.4916 & 0.4429 \\ -0.49169 & 0.5965 & -0.3889 \\ 0.4429 & -0.3889 & 0.3639 \end{bmatrix}$	17.5922 63.8137 110.8417	0.0044 0.0011 0.0025
2	$C = \begin{bmatrix} 0.7507 & -0.4704 & 0.4132 \\ -0.4704 & 0.5908 & -0.3695 \\ 0.4132 & -0.3695 & 0.3403 \end{bmatrix}$	17.5922 63.8138 110.8417	0.0045 0.0011 0.0026
3	$C = \begin{bmatrix} 1.1581 & -0.5269 & 0.6317 \\ -0.5269 & 0.4586 & -0.3857 \\ 0.6371 & -0.3857 & 0.4663 \end{bmatrix}$	17.5920 63.8138 110.8416	0.0067 0.0011 0.0024
4	$C = \begin{bmatrix} 1.1186 & -0.5359 & 0.6131 \\ -0.5359 & 0.4538 & -0.3888 \\ 0.6131 & -0.3888 & 0.4493 \end{bmatrix}$	17.5921 63.8138 110.8416	0.0056 0.0011 0.0024

Table 6
Damping matrix adjusted for different methods considering the complex eigenvectors (clamped plate model)

Method	Damping matrix	Frequency [Hz]	Damping ratio
1	$C = \begin{bmatrix} 0.7821 & -0.4916 & 0.4429 \\ -0.49169 & 0.5965 & -0.3889 \\ 0.4429 & -0.3889 & 0.3639 \end{bmatrix}$	17.5922 63.8137 110.8417	0.0044 0.0011 0.0025
2	$C = \begin{bmatrix} 6.8598 & -1.8092 & 3.3693 \\ -1.8092 & 3.8442 & -3.6450 \\ 3.3693 & -3.6450 & 1.7371 \end{bmatrix}$	17.6160 63.7691 110.7709	0.0012 0.0013 0.0022
3	$C = \begin{bmatrix} -6.5648 & 1.6231 & -3.8587 \\ 1.6231 & 3.1434 & 0.2908 \\ -3.8587 & 0.2908 & -2.1174 \end{bmatrix}$	17.5993 63.8124 110.7944	0.0103 0.0015 0.0033
4	$C = \begin{bmatrix} -6.4220 & 1.7625 & -3.7741 \\ 1.7625 & 3.0750 & 0.3787 \\ -3.7741 & 0.3787 & -2.0763 \end{bmatrix}$	17.6001 63.8124 110.8417	0.0044 0.0014 0.0029

Table 7
Modal parameters (transmission line cable model)

	Mode number	Frequency [Hz]	Damping ratio
Numeric	1	2.1609	
RFPM		2.1568	0.0031
Numeric	2	4.3219	
RFPM		4.2931	0.0017
Numeric	3	6.4830	
RFPM		6.4116	0.0015

As it was used three proximity sensors in the positions cited previously, the reduced system has order of 3×3. The Tables 8 and 9 show the damping matrices estimated with different methods.

Table 8
Damping matrix adjusted for different methods considering only the real part of the eigenvectors (transmission line cable model)

Method	Damping matrix	Frequency [Hz]	Damping ratio
1	$C = \begin{bmatrix} 8.7283 & -14.4401 & 9.1571 \\ -14.4401 & 25.9016 & -16.5308 \\ 9.1571 & -16.5308 & 11.2800 \end{bmatrix}$	2.1645 4.3209 6.4830	0.0036 0.0018 0.0015
2	$C = \begin{bmatrix} 8.4634 & -14.2311 & 8.9913 \\ -14.2311 & 25.8552 & -16.4127 \\ 8.9913 & -16.4127 & 11.2071 \end{bmatrix}$	2.1645 4.3209 6.4830	0.0036 0.0018 0.0015
3	$C = \begin{bmatrix} 9.0835 & -14.8468 & 9.4339 \\ -14.8468 & 26.3387 & -16.8580 \\ 9.4339 & -16.8580 & 11.4774 \end{bmatrix}$	2.1645 4.3209 6.4830	0.0036 0.0018 0.0016
4	$C = \begin{bmatrix} 8.9961 & -14.7154 & 9.3373 \\ -14.7154 & 26.1350 & -16.7091 \\ 9.3373 & -16.7091 & 11.3628 \end{bmatrix}$	2.1645 4.3209 6.4830	0.0036 0.0018 0.0016

Table 9
Damping matrix adjusted for different methods considering the complex eigenvectors (transmission line cable model)

Method	Damping matrix	Frequency [Hz]	Damping ratio
1	$C = \begin{bmatrix} 8.7283 & -14.4401 & 9.1571 \\ -14.4401 & 25.9016 & -16.5308 \\ 9.1571 & -16.5308 & 11.2800 \end{bmatrix}$	2.1645 4.3209 6.4830	0.0036 0.0018 0.0015
2	$C = 1000 \times \begin{bmatrix} 0.5711 & -0.9710 & 0.6305 \\ -0.9710 & 1.6219 & -1.0602 \\ 0.6305 & -1.0602 & 0.6540 \end{bmatrix}$	2.2029 4.2983 6.3990	0.0351 0.0151 0.0014
3	$C = \begin{bmatrix} -173.6554 & 284.3900 & -178.5707 \\ 284.3900 & -453.0179 & 268.6563 \\ -178.5707 & 268.6563 & -146.8172 \end{bmatrix}$	2.1665 4.3272 6.4671	0.0137 0.0029 0.0039
4	$C = \begin{bmatrix} -168.6580 & 279.0105 & -171.5325 \\ 279.0105 & -450.4612 & 261.8025 \\ -171.5325 & 261.8025 & -138.3978 \end{bmatrix}$	2.1667 4.3272 6.4671	0.0043 0.0010 0.0030

5. Conclusions

The rational fraction polynomial method (RFPM) was used to modal analysis of the systems. It is possible to notice in Tables 1, 4 and 7 that the experimental modal results present good concordance with the numeric values.

As the analyzed systems are lightly damped, the modal frequencies are little influenced by the adjusted damping matrices. However, the damping ratio is very influenced by these matrices.

The first model (two bars in balancing) presented the better results. All the results (methods 1–4) in relation to the damping ratio and the natural frequency are close to the experimental values. This fact was expected once that the system is small and all the physical parameters were easily estimated.

The second model (clamped plate) is very influenced by imaginary part of the eigenvectors. When it is considered only the real part of the eigenvectors, the results presented good agreement between numeric and experimental results.

The third model (transmission line cable) also presented not satisfactory results when the imaginary part of the eigenvectors was considered. Like the amplitude of the imaginary part of eigenvectors is very smaller than the real part, the experimental data can be contaminated by noise. This fact apparently is the main limitation for application of the methods.

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